Find the equation of the normal line to the following curve:

$y = x^2 - 4x + 2$ **at x = 3**

$$\lim_{x\to 3} \frac{f(x) - f(3)}{x-3}$$

$$\lim_{x\to 3} \frac{x^2-4x+2-(-1)}{x-3}$$

$$\lim_{x\to 3} \frac{x^2-4x+3}{x-3}$$

$$\lim_{x\to 3} \frac{(x-3)(x-1)}{x-3}$$

$m = 2$

$f(3) = 3^2 - 4(3) + 2$

$= -1$

Pt $(3, -1)$

$m = 2$

$m_{normal} = -\frac{1}{2}$

$y = mx + b$

$-1 = -\frac{1}{2}(3) + b$

$-1 = -\frac{3}{2} + b$

$\frac{1}{2} = b$

$y = -\frac{1}{2}x + \frac{1}{2}$

Calculus 120
Unit 1: Rate of Change and Derivatives

February 7, 2019: Day #6

1. Assignments and Quiz Returned and Discussed

2. DERIVATIVES

Curriculum Outcomes

C1. Explore the concepts of average and instantaneous rate of change.

Derivatives

Remember our formula for slope of a tangent line at a given point....

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

What if we had to find the slopes of numerous tangent lines at different points for the same function? We would need to apply the above formula a number of times. And often, we end up with limits that cannot be calculated easily algebraically. This concept has so many applications in mathematics, that the formula has been generalized so that we can find the slopes of all tangents to a function. This new, generalized formula is known as The Definition of a Derivative or First Principles of Calculus.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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We can think of derivatives in two ways:

1. The derivative at a point is a numerical value which represents the slope of a tangent line at a given point or the instantaneous rate of change at a given point. The process we have been using finds the derivative at a point.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

2. We can also think of the derivative as a function. The derivative is a special new function that can be used to find slopes of tangent lines for given functions at any point on the curve. As a result, the derivative is often known as the slope function of a curve.

The process of finding a derivative is known as differentiation.

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

→ - Defintion of a derivative
- First Principles
- Slope function

There are numerous symbols that can be used to denote derivatives. The most common are as follows...

y' pronounced "y prime"

$f'(x)$ pronounced "f prime of x"

$\frac{dy}{dx}$ pronounced "the derivative of y with respect to x" or just "d-y-d-x"

Example:

a) If $f(x) = x^2$, find $f'(x)$.

$f(2)$ $f(0)$ $f(\pi)$ $f(8)$

b) Find the instantaneous rate of change of f(x) at x = 1, x = 2, x = 3, x = 4, and x = 5.

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$(x+h)^2$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$x^2 + 2xh + h^2$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= \lim_{h \to 0} \frac{h(2x+h)}{h}$

$f'(x) = 2x$

b) $f'(1) = 2$

$f'(2) = 4$

$f'(3) = 6$

$f'(4) = 8$

$f'(5) = 10$

If y = 2x² - 5x + 6, find y' and the equation of the tangent line to y at x = 3.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{2(x+h)^2 - 5(x+h) + 6 - (2x^2 - 5x + 6)}{h}$$

$$\lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 5x - 5h + 6 - 2x^2 + 5x - 6}{h}$$

$$\lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h + 6 - 2x^2 + 5x - 6}{h}$$

$$\lim_{h\to 0} \frac{4xh + 2h^2 - 5h}{h}$$

$$\lim_{h\to 0} \frac{h(4x + 2h - 5)}{h}$$

$$f'(x) = 4x - 5$$

$f'(3) = 4(3) - 5$

$m = 7$

$f(3) = 2(3)^2 - 5(3) + 6$
$= 18 - 15 + 6$
$= 9$

$(3, 9)$

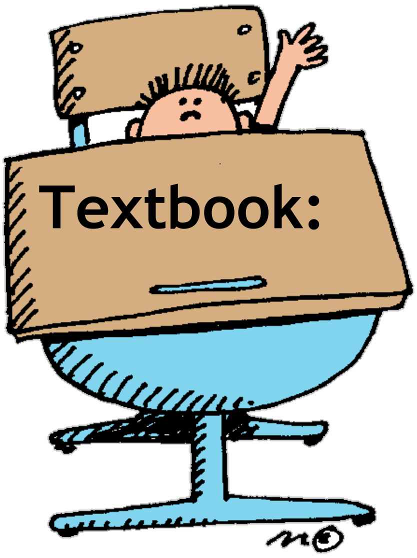
$y = 7x - 12$

$m = 7$
$P(3, 9)$

$y = mx + b$
$9 = 7(3) + b$
$9 = 21 + b$
$-12 = b$

Ex:

Find y' and the slope of the tangent line at $x = -5$ for $f(x) = x^3 + x + 2$



Practice

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#1, 3, 7, 9, 10, 11, 12, 17, 18, 19

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Attachments

2.1_74_AP.html

2.1_74_AP.swf

2.1_74_AP.html